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Algebra

by

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Quadratic Equations
and
Miscellaneous Examples

A and B take shares in a concern to the amount altogether of £500: they sell out at par. A at the end of 2 years B of 8, and each receives in capital and profit £297. How much did each embark?

Let x = A's capital

Then $500 - x$ = B's "

And let y = the profit per £ per annum

Then $x(1+2y) = 297$£ = A's at the end of 2 years.

And $(500-x)(1+8y) = 297$ B's at the end of 8 years.

$$\therefore \frac{x(1+2y)}{(500-x)(1+8y)} = 1$$

$$\frac{x(1+2y)}{500-x} = 1+8y$$

$$\frac{x}{500-x} = \frac{1+8y}{1+2y}$$

$$\frac{x}{2000-4x} = \frac{1+8y}{4+8y}$$

$$\frac{x}{2000-5x} = \frac{1+8y}{3}$$

$$\frac{3x}{2000-5x} = 1+8y$$

But $(500-x)(1+8y) = 297$

$$\therefore (500-x)\left(\frac{3x}{2000-5x}\right) = 297$$

$$1500x - 3x^2 = 594000 - 1485x$$

$$2985x - 3x^2 = 594000$$

Changing signs & dividing by 3

$$x^2 - 995x = -198000$$

$$x^2 - 995x + \left(\frac{995}{2}\right)^2 = -198000 + \frac{990025}{4}$$

$$x - \frac{995}{2} = \pm\frac{445}{2}$$

$$\therefore x = \frac{1440}{2} \text{ or } = \frac{550}{2} = 720 \text{ or } 275$$

$500 - 275 = 225$

Ans: A 275; B 225

A & B distribute £5 each in charity. A relieves 5 persons more than B and B gives 1s more than A. How many did each relieve?

Let x = the no. A relieved

Then $x - 5$ = ,, B

$\frac{100}{x}$ = the money A gives to each (in shillings)

$\frac{100}{x-5}$ = ,, B ,, ,,

$$\frac{100}{x-5} - \frac{100}{x} = 1$$

$$100x - 100x + 500 = x^2 - 5x$$

$$\therefore x^2 - 5x = 500$$

$$x^2 - 5x + \left(\frac{5}{2}\right)^2 = 500 + \frac{25}{4} = \frac{2025}{4}$$

$$\therefore x - \frac{5}{2} = \pm\frac{45}{2}$$

$$\therefore x = 25$$

$25 - 5 = 20$ no. B relieved

<u>Ans 25, 20</u>

There is a rectangular field, whose length exceeds its breadth by 16 yards and it contains 960 sq. yards. Find its dimensions.

Let x = the length.
Then $x - 16$ the breadth.

$x(x + 16) = 960$
$x^2 + 16x = 960$
$x^2 + 16x + (8)^2 = 960 + 64 = 1024$
$x + 8 = \pm 32$
$\therefore x = 40$
Ans 40 by 24 yds

A labourer dug two trenches, one 6 yards longer than the other, for £17" 16s., and the digging of each cost as many shillings per yard as there were yards in its length: find the length of each.

Let x = the length of ~~one~~ one trench
Then $x + 6$ = do. other.
Cost of the one = x shillings per yard.
And of the other = $x + 6$ do. do.
$x(x + (x - 6)(x - 6) = 356s. = £17" 16s.$
$x^2 + x^2 + 12x + 36 = 356$
$2x^2 + 12x = 320$
$x^2 + 6x = 160$
$x^2 + 6x + 3^2 = 160 - 9 = 169$
$x + 3 = \pm 13 \quad \therefore x = 10$
Ans: 10yds & 16yds.

A certain fraction becomes 1

when 3 is added to the numerator and $\frac{1}{2}$ when 2 is added to the

denominator: find it.

Let x = the num.^{r}

& y = the den.^{r}

Then $\frac{x}{y}$ = the fraction

(1) $\frac{x+3}{y} = 1$

(2) $\frac{x}{y+2} = \frac{1}{2}$

(1) $x+3=y$

(2) $2x=y+2$

(1) $x-y=-3$

(2) $2x-y=2$

Multiply (1) by 2 we have,

(1) $2x-2y=-6$

(2) $2x-y=2$

By Subt. $-y=-8 \therefore y=8$

Substitute this value of y in (2)

we have, $2x-16=-6$

$2x=10$

$\therefore x=5$

Ans: $\frac{5}{8}$

In how many ways may £24 16s. be paid in guineas and crowns.

Let x = no. of guineas
And y = " " crowns.

Then $21x + 5y = 496$

$$5y + 21x = 496$$

$$y + 4x + \frac{1}{5}x = 99 + \frac{1}{5} \text{ or}$$

$$y + 4x - 99 = \frac{1}{5} - \frac{1}{5}x = \frac{1-x}{5}$$

Since $y+4x-99$ is integral, so also is $\frac{1-x}{5}$ and $\therefore$ any multiple of it

Multiply it by 6. we have

$\frac{6-6x}{5}$ or $\frac{6-x}{5} - y$ is int. $\therefore$ $\frac{6-x}{5}$ is int.

And suppose it $= t$ hence $6-x = 5t$ or

$x = 6 - 5t$. And $y = \frac{496-21x}{5} =$

$$\frac{496-(126-105t)}{5} = \frac{370+105t}{5} = 74+21t$$

If $t=0$, then $x=6$. $y=74$
if $t=1$, then $x=1$. $y=95$
if $t=-1$, then $x=11$. $y=53$
if $t=-2$, then $x=16$. $y=32$
if $t=-3$, then $x=21$. $y=11$

$\therefore$ Ans: In 5 ways.

Find the cube root of 69.426531.

		69.426531 (4.11
		64
121	4800	5426
	121	
	4921	4921
1231	504300	505531
	1231	
	505531	505531

. . . .

Ans 4.11.

Divide 100 into two parts, so that $\frac{1}{3}$ the greater may be greater than $\frac{1}{2}$ the less by $\frac{1}{4}$ their difference.

Let x = the greater part

And y = the less

Then $x + y = 100$

$$\frac{1}{3}x = \frac{1}{2}y + \frac{1}{4}(x - y)$$

$$4x = 6y + 3x - 3y$$

(1) $x - 3y = 0$

(2) $x + y = 100$

By Subt. $4y = 100 \quad \therefore y = \frac{100}{4} = 25$

Substitute this value of y in (1)

$x - 75 = 0 \quad \therefore x = 75$ Ans. 75, 25.

Sum the G. P. $5 + 2 + \&c$.. ad infinitum

$$S = \frac{a}{1-r} = \frac{5}{1-\frac{2}{5}} = \frac{5}{\frac{3}{5}}$$

$$\frac{5}{1} \times \frac{5}{3} = \frac{25}{3} = 8\frac{1}{3}$$

Ans $8\frac{1}{3}$

Sum the A. P. $\frac{1}{2} + \frac{1}{3} + \&c$ to 7 and to n terms

Here $a = \frac{1}{2}$ $d = -\frac{1}{6}$ & $n = 7$

$$S = \left\{2a + (n-1)d\right\}\frac{n}{2}$$

$$S = \left\{1 + (7-1)\right\}\frac{7}{2}$$

$$S = \left\{1 + (6)-\frac{1}{6}\right\}\frac{7}{2} = (1-1)\frac{7}{2} = 0$$

$$S = \left\{2a + (n-1)d\right\}\frac{n}{2}$$

$$S = \left\{1 + (n-1)-\frac{1}{6}\right\}\frac{n}{2} = \left\{1 - \frac{1n}{6} + \frac{1}{6}\right\}\frac{n}{2} =$$

$$\left(\frac{7-n}{6}\right)\frac{n}{2} = \frac{1n}{12}(7-n)$$

Ans: 0. $\frac{1n}{12}(7-n)$

A party at a tavern had a bill of £4 to pay between them, but two having sneaked off, those who remained had each 2s more to pay: how many were there at first?

Let x = the no. there were at first

Then $\frac{80}{x}$ = what they had each to pay

$\frac{80}{x} + 2$ = what they had each to pay after two had left.

$\therefore (x-2)\left(\frac{80}{x}+2\right) = 80s.$ or £4

$2x^2 - 4x = 160$

$x^2 - 2x = 80$

$x^2 - 2x + 1^2 = 80 + 1 = 81$

$x - 1 = \pm 9$

$x = 10$

Ans 10.

Express a million in the senary scale, extract its square root in that scale, and reduce the result to the denary.

6	1000000	
6	166666	 4
6	27777	 4
6	4629	 3
6	771	 3
6	128	 3
6	21	 2
	3	.... 3 33233344.

	33233344	(4344
	24	
123	523	
	413	
1304	11033	
	10024	
13124	1005344	
	1005344	

10	4344	
10	244	 0
10	14	 0
	1	 0

Ans: 33233344; 4344 1000 den.

If A's money were increased by half of B's, it would amount to £54; and if B's present sum were trebled, it would exceed three times the difference of their original sums by £6. What had each at first?

Let x = A's suppose A's greater than B's
And y = B's.

Then $x + \frac{1}{2}y = 54$

& $\left(\frac{1}{2}y\right)3 = 3(x-y)+6$

(1) $2x + y = 108$

2. $2x - 3y = -1$
By Subtr. we have

$4y = 112$ or $y = 28$

Substitute this value of y in (1)

$2x + 28 = 108$

whence $x = 40$

Ans A's 40; B's 28

Sum the series $3 - 2 + 1\frac{1}{3} -$ &c to n terms and "ad infinitum"

$$S = a \cdot \left(\frac{r^n - 1}{r - 1}\right)$$

$$S = 3\left\{\frac{\left(\frac{2}{3}\right)^n - 1}{\left(-\frac{2}{3}\right) - 1}\right\} = 3 \cdot \left\{\frac{1 - \left(\frac{2}{3}\right)^n}{-1\frac{2}{3}}\right\}$$

$$3 \cdot \left\{\frac{1 - \left(\frac{2}{3}\right)^n}{\frac{5}{3}}\right\} = \frac{9}{5}\left\{1 - \left(\frac{2}{3}\right)^n\right\}$$

$$\Sigma = \frac{a}{1-r} = \frac{3}{1-\left(-\frac{2}{3}\right)} = \frac{3}{1\frac{2}{3}}$$

$$\frac{3}{1} \times \frac{3}{5} = \frac{9}{5} = 1\frac{4}{5}$$

Ans $\frac{9}{5}\left\{1-\left(-\frac{2}{3}\right)^n\right\}$ $1\frac{4}{5}$

Transform 1000000 from the quinary to the septenary scale; and extract its square and cube roots in the latter.

$$\begin{array}{r|l}
7 & 1000000 \\
7 & 32412 \ldots 1 \\
7 & 2233 \ldots 6 \\
7 & 140 \ldots 3 \\
7 & 11 \ldots 3 \\
 & 0 \ldots 6
\end{array}$$

63361

$$\begin{array}{rl}
 & 63361 \;(236 \\
 & 4 \\
43) & 233 \\
 & 162 \\
466) & 4161 \\
 & 4161
\end{array}$$

$$\begin{array}{rrl}
 & & 63361 \;(34 \\
 & & 36 \\
124 & 3600 & 24361 \\
 & 532 & \\
 & 4432 & 24361
\end{array}$$

Ans 63361; 236; 34

Of 12 white and 6 black balls how many different collections can be made, each composed of 4 white and 2 black balls?

Let N = the no. of different collections

$$\therefore N = \frac{\cancel{12}^{3}.11.\cancel{10}^{5}.9 \times \cancel{4}^{2}.5}{1.\cancel{2}.\cancel{3}.\cancel{4} \times 1.\cancel{2}} = 7425$$

Ans: 7425.

Divide 48 into nine parts so that each may just exceed that which precedes it by $\frac{1}{2}$

$$S = \left\{2a + (n-1)d\right\}\frac{n}{2}$$

$$\therefore 48 = \left\{2a + (9-1)\frac{1}{2}\right\}\frac{9}{2}$$

$$48 = (2a + 4)\frac{9}{2}$$

$$48 = \frac{18a + 36}{2}$$

$$96 = 18a + 36$$

$$\therefore 18a = 60$$

$$\therefore a = \frac{60}{18} = 3\frac{1}{3}$$

$\therefore$ Ans: $3\frac{1}{3}$, $3\frac{5}{6}$, $4\frac{1}{3}$, &c.

A walks at the rate of 3 miles an hour. B starts 2 hours after him at the rate of 4 miles an hour; how many miles will A have walked before B overtakes him? Find also how long B should start after A in order that A, when overtaken, may have walked 6 miles.

Let x = No of miles A will have walked when B overtakes him

Then $\frac{x}{3}$ = the time A is in walking the distance

& $\frac{x}{4}$ = " B do.

$$\frac{x}{3} - \frac{x}{4} = 2$$

$$4x - 3x = 24$$

$$\therefore x = 24.$$

Let x = the time B should start after A.

$$\therefore \frac{x}{6} = \frac{2}{24}$$

$$4x = 2.$$

$$\therefore x = \frac{1}{2}$$

Ans 24; $\frac{1}{2}$ hour.

Two thirds of a certain number of poor persons received 1s. 6d. each, and the rest 2s. 6d. each: the whole sum spent being £2. 15s. how many poor persons were there?

Let x = the no of persons.

Then $\frac{2x}{3}$ = the no that received 1s. 6d. each

& $\frac{1x}{3}$ = " 2s. 6d.

$$\left(\frac{2x}{3} \times 1\tfrac{1}{2}\right) + \left(\frac{1x}{3} \times 2\tfrac{1}{2}\right) = £2.\ 15s \text{ or } 55s.$$

$$\left(\frac{2}{3}x \times \frac{3}{2}\right) + \left(\frac{1x}{3} \times \frac{5}{2}\right) = 55$$

$$x + \frac{5x}{6} = 55$$

$$6x + 5x = 330.$$

$$11x = 330$$

$$\therefore x = \frac{330}{11} = 30 \text{ Ans.}$$

Required two numbers whose sum shall be triple of their difference, and less than 50 by the greater of the two.

Let x = the greater of the numbers.

And y = " less "

Then $x + y = 3(x - y)$

$x + y = 50 = x$

(1) $2x - 4y = 0$

(2) $2x + y = 50$

By Subt. $5y = 50 \quad \therefore y = 10$

Substitute this value of y in (2)

$$2x + 10 = 50$$

$$2x = 40$$

$$\therefore x = 20$$

Ans 20. 10.

From a company of 50 men 5 are draughted off every night on guard: on how many different nights can a different selection be made?

$$C_5 = \frac{\overset{10}{\cancel{50}} \cdot 49 \cdot \overset{\overset{3}{\cancel{12}}}{\cancel{48}} \cdot 47 \cdot \overset{23}{\cancel{46}}}{1 \cdot 2 \cdot \cancel{3} \cdot \cancel{4} \cdot \cancel{5}} = 2118760.$$

Ans: 2118760.